the particular reciprocal lattice vector K used; the latter depends on the initial state k.

Figure 4-2 shows a cross section of the Brillouin zone for a b.c.c. lattice, taken in a 001 plane. The circles are cross sections of the Fermi surface and the dashed square is a zone for phonons, centered on the state $\vec{k}$. The U.K. processes will be those for which the final state k' lies on that portion of the Fermi sphere centered at 0 which is outside the dashed square. A typical U.K. process and the associated phonon vector $\vec{q}$ is shown. For fixed θ, the length |q| depends on the initial state $\vec{k}$; this is most clearly seen by taking θ = 180° and $\vec{k}$ first in the 100 and then in the 110 direction. For the latter direction |q| is about 1/4 as large as for k in the 100.

Normal processes must use longitudinal phonons, at least for those directions in which a separation into longitudinal and transverse modes is possible, because the term $\hat{e}_{q,p} \cdot \hat{s}$ becomes $\hat{e}_{q,p} \cdot \hat{q}$ for a normal process and this is zero for transverse modes. On the other hand, for an U.K. process $\vec{s}$ is not usually parallel to $\vec{q}$ and transverse phonons may participate; in fact the low value of the velocity of sound for transverse phonons will act as a weighting factor which emphasizes those U.K. processes which use transverse phonons.

One source of anisotropy in τ is then the k dependence of the contribution of $1/q^2$ to the integral for $\tau(\vec{k})$. This source occurs in any b.c.c. metal; for the case of the alkali metals the anisotropy of the velocity of sound must also be considered.

We now wish to make some estimate of the anisotropy of $\tau(\vec{k})$. The correct way to obtain $\tau(\vec{k})$ would be to choose a direction $\vec{k}$ and compute q, $c^2_{\hat{q},p}$, and $e_{\hat{q},p}$ for a large number of points k' on a Fermi sphere and evaluate Eq. (IV-37). This would be a major computational task. A simpler but considerably less accurate procedure would be to consider only scattering in two dimensions and evaluate the θ integral in Eq. (IV-37) for fixed ϕ. Some of the loss of accuracy comes from the fact that for some k directions the θ integral depends strongly on the value of ϕ chosen. For example, if one chose k in the 110 direction and replaced the Brillouin zone for phonons

by a sphere of equal volume, there is ϕ symmetry in the sense that the $|\vec{q}|$ for fixed θ is independent of ϕ, although $c_{\hat{q},p}$ and $\hat{e}_{\hat{q},p} \cdot \hat{s}$ are not; on the other hand, for k in a 111 direction this is not true. In a repeated zone scheme the nearest neighbor Fermi spheres do not have their centers on a 111 axis and consequently the section of these spheres cut by the plane containing k (111) and k' depends on ϕ. This makes it difficult to estimate τ (111) by doing only the θ integration.

We decided to estimate the anisotropy in $\tau (\vec{k})$ by evaluating

$$I(\vec{k}) = \int_{o}^{2\pi} \frac{(1 - \cos \theta) |\sin \theta| [(JS) \theta]^2 d\theta}{|\vec{q}|^2} \qquad (IV-40)$$

for $\vec{k}$ in the 100, 110, and 111 directions. This is a very crude procedure, which not only replaces the three dimensional integration in Eq. (IV-37) by a two dimensional one, but also considers the velocity of sound, the polarization factor $\hat{e}_{\hat{q},p} \cdot \hat{s}$, and the density of states as constants. Although we could have approximated $c^2_{\hat{q},p}$ in Eq. (IV-37) by using the velocity of sound in the principal direction closest to that of $\vec{q}$, we felt this procedure ran the risk of weighting the integral by the sound velocity for an unrepresentative phonon; it would also give a factor $\frac{1}{c^2_{\hat{q},p}}$ that was discontinuous in θ.

We obtained the values of $(JS)^2$ given by Bailyn [13]; in Table IV-4 we list his values of $u^3 (JS)^2$ vs. $u = \frac{\sin \theta}{2}$ for potassium and lithium, The square of the matrix element for potassium has values less than those for rubidium and cesium and greater than those for sodium; the general behavior of $(JS)^2$ vs. u is similar for all these four metals and we chose potassium as representative of them. Lithium is unlike the other alkalis in that $(JS)^2$ goes through a zero near $\theta = 120°$. In Fig. 4-3 we have plotted

$$F(\theta) = (JS)^2 (1 - \cos \theta) \sin \theta \qquad (IV-41)$$

for θ from 0 to $180°$ in a polar plot, using Bailyn's values for the potassium and lithium matrix elements. For θ from $180°$ to $360°$ the absolute value of $\sin \theta$ should be used. $F(\theta)$ peaks rather sharply near $75°$ for lithium; this